§1.0 POLYNOMIAL FUNCTIONS – Prerequisite Skills

Function Notation

- The notation $f(x)$ means that we have a function named *f* and the variable in that function is *x*.
- The domain of the function describes values of *x* that can be put into the function.
- The range of the function describes values that $f(x)$ can equal.

Examples:

1. Determine each value for the function $f(x) = -5x - 9$.

a) $f(2)$ **b)** $f(-4)$

c) $f(-3x)$ **d)** $f(m + 1)$

2. Determine each value for the function $f(x) = -x^2 + 4x - 3$.

a) $f(-1)$ **b)** $f(0)$

c) $f(-2x)$ **d)** $2f(m - 1)$

Slope and Y-Intercept of a Line

- Standard Form: Equations of the form $Ax + By + C = 0$
- Slope – Intercept Form: Equations of the form $y = mx + b$
- To switch from standard form to slope – intercept form means you need to solve for y.

Examples:

3. State the slope and the *y*-intercept of each line

a) $y = -4x - 1$ **b)** $3y = 9x + 2$

c) $2x - 4y + 12 = 0$ **d)** $x + 2y = 6(x - 1)$

Equation of a Line

- To find the equation of a line you need the slope and y-intercept.

Examples:

4. Determine an equation for the line that satisfies the following conditions.

a) The slope is $\frac{2}{5}$ and the *y*-intercept is –2.

b) The slope is $\frac{1}{3}$ and the line passes through the point (6, –2).

c) The line passes through the points (–3, 4) and (2, –1).

Finite Differences

- Remember that the first set of finite differences for a linear function will be constant.
- Remember that the second set of finite differences for a quadratic function will be constant.

Examples:

5. Use finite differences to determine if each function is linear, quadratic, or neither.

a)

x	*y*
–2	0
–1	–4
0	–6
1	–6
2	–4
3	0

b)

x	*y*
–3	–28
–2	0
–1	10
0	8
1	0
2	–8

Domain and Range

- The **domain** of a function is the complete set of possible values of the independent variable in the function.
    - In plain English, this definition means: The domain of a function is the set of all possible ***x*** values which will make the function "work" and will output real y-values.
    - When finding the domain, remember:
        - The denominator (bottom) of a fraction cannot be zero.
        - The values under a square root sign must be positive.
- The **range** of a function is the complete set of all possible resulting values of the dependent variable of a function, after we have substituted the values in the domain.
    - In plain English, the definition means: The range of a function is the possible y values of a function that result when we substitute all the possible x-values into the function.
    - When finding the range, remember:
        - Substitute different x-values into the expression for y to see what is happening.
        - Make sure you look for minimum and maximum values of y.
        - Draw a sketch!

<u>Examples</u>:

6. State the domain and range of each function.

a) $y = -3(x - 2)^2 - 1$ **b)** $y = \sqrt{-3x-4}$

Quadratic Functions

- Standard Form: $ax^2 + bx + c$, where $a \neq 0$
- Vertex Form: $y = a(x - h)^2 + k$, where $a \neq 0$
    - Vertex, V = (h, k)
    - D: $\{ x \in \Re \}$
    - R: $\{ y \in \Re \mid y \leq k \}$ when $a < 0$ OR R: $\{ y \in \Re \mid y \geq k \}$ when $a > 0$
- Factored Form: $y = a(x - s)(x - r)$, where $a \neq 0$
    - Zeroes: x = s and x = r

<u>Examples</u>:

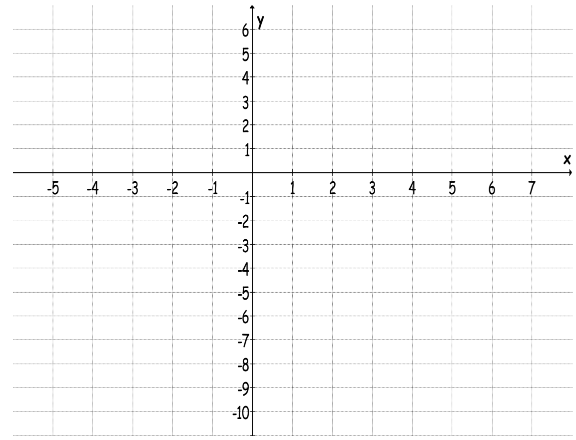
7. Determine the equation of a quadratic function that has y-intercept 4 and x-intercepts –2 and 3.

8. Determine the x-intercepts, the vertex, the direction of opening, and the domain and range of each quadratic function. Then, graph each function.

a) $y = 3(x - 1)^2 - 3$

b) $y = (x - 2)(x + 4)$

c) $y = -0.5(x + 1)(x - 5)$



Transformations

- Shifting
  - Given a function y = f(x) and a constant c > 0
    - y = f(x) + c shifts the graph up c units (add c to y-values)
    - y = f(x) − c shifts the graph down c units (subtract c from y-values)
    - y = f(x + c) shifts the graph left c units (subtract c from x-values)
    - y = f(x − c) shifts the graph right c units (add c to x-values)
- Stretching & Compressing
  - Given a function y = f(x) and a constant c > 1
    - $y = c \cdot f(x)$ vertical stretch by a factor of c (multiply y-values by c)
    - $y = \frac{1}{c} \cdot f(x)$ vertical compress by a factor of $\frac{1}{c}$ (divide y-values by c)
    - $y = f(c \cdot x)$ horizontal compress by a factor of $\frac{1}{c}$ (divide x-values by c)
    - $y = f(\frac{1}{c} \cdot x)$ horizontal stretch by a factor of c (multiply x-values by c)
- Reflecting
  - Given a function y = f(x)
    - y = −f(x) reflects graph about the x-axis (multiply all y values by −1)
    - y = f(−x) reflects graph about the y-axis (multiply all x values by −1)

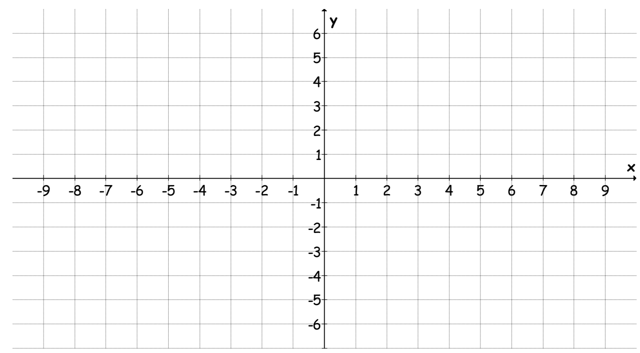
<u>Examples</u>:

9. Describe each transformation that must be applied to the function $y = f(x)$.

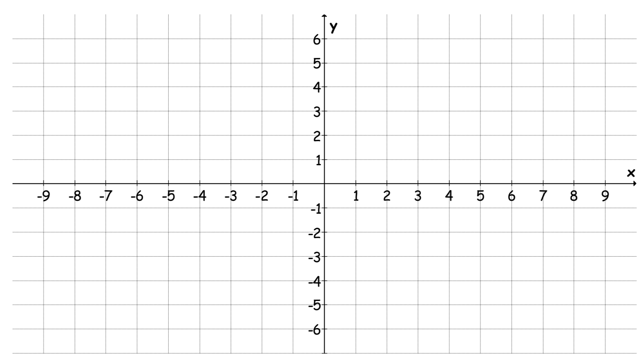
a) $y = 2f(x - 5)$ **b)** $y = f(x + 1) - 4$ **c)** $y = -f(3x) + 1$

10. i) Write an equation for the transformed function of each base function.
ii) Sketch a graph of each function.
iii) State the domain and range of each function.

a) $f(x) = x$ is compressed vertically by a factor of $\frac{1}{3}$, reflected in the x-axis and translated 4 units to the right.



b) $f(x) = x^2$ is stretched vertically by a factor of 3, compressed horizontally by a factor of $\frac{1}{2}$ and translated 6 units to the left and 5 units down.



11. Describe the transformation that must be applied to the base function $y = x^2$ to obtain the graph of the function

$$y = -\left(\frac{1}{2}x - 4\right)^2 + 3.$$